University of California, Berkeley
Physics 105 Fall 2000 Section 2 (*Strovink*)

# PRACTICE EXAMINATION 1

**Directions.** Do all problems (weights are indicated). This is a closed-book closed-note exam except for one $8\frac{1}{2} \times 11$ inch sheet containing any information you wish on both sides. You are free to approach the proctor to ask questions – but he will not give hints and will be obliged to write your question and its answer on the board. Roots, circular functions, *etc.*, may be left unevaluated if you do not know them. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Box or circle your answer.

**1.** (20 points)
In one dimension, the Lagrangian for a relativistic free electron is

$$\mathcal{L} = -mc^2\sqrt{1 - \frac{\dot{x}^2}{c^2}} \ ,$$

where $m$ is the electron mass and $c$ is the speed of light.

Find the total energy of the electron in terms of $m$, $c$, and $\dot{x}$. Prove your result given only the Lagrangian, using no other knowledge of relativity.

**2.** (25 points)
During $-\infty < t < 0$, a linear oscillator satisfying the equation of motion

$$\ddot{x} + \omega_0 \dot{x} + \omega_0^2 x = \frac{F_x(t)}{m}$$

is driven at its resonant frequency by a force per unit mass

$$\frac{F_x(t)}{m} = a_0 \cos \omega_0 t \ ,$$

where $a_0$ is a constant.
(**a**) (10 points)
Find $x(0)$ and $\dot{x}(0)$ at $t = 0$.
(**b**) (15 points)
At $t = 0$ the driving force is turned off. Find $x(t)$ for $t > 0$.

**3.** (35 points)
A small bead of mass $m$ is constrained to move without friction on a circular hoop of radius $a$ that rotates with constant angular velocity $\Omega$ about a vertical diameter. Use $\theta$, the polar angle of the bead, as the single generalized coordinate ($\theta = 0$ at the bottom). Do not neglect gravity.
(**a**) (5 points)
Write the Lagrangian as a function of $\theta$ and $\dot{\theta}$. Remember to take into account the two different components of the bead's velocity.
(**b**) (5 points)
Obtain the differential equation of motion for $\theta$.
(**c**) (10 points)
Find a restriction on $\Omega$ such that small oscillations about $\theta = 0$ can occur. What is the angular frequency of these oscillations?
(**d**) (5 points)
If $\Omega$ does not obey the restriction in part (**c**), about what other equilibrium position(s) can the bead undergo small oscillations?
(**e**) (10 points)
What is the angular frequency of the small oscillations to which part (**d**) refers?

**4.** (20 points)
An antiproton with mass $m$ and charge $-e$ is incident upon a nucleus with mass $M$ and charge $Ze$. You may assume them to be point particles. The nucleus is initially at rest. When $m$ is still very far from $M$, it has velocity $v_0$, directed so that the two masses would miss by a distance $b$ if there were no attraction between them. *State the system of units in which you are working (SI or cgs).*
(**a**) (10 points)
Obtain a pair of equations which, if solved, would allow you to calculate the distance of closest approach between the antiproton and the nucleus.

(**b**) (10 points)
As $v_0$ approaches zero, through what angle will the antiproton scatter? (Elementary reasoning, if stated correctly, should be sufficient here.)